UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

FURTHER MATHEMATICS **9231/23**

Paper 2 **October/November 2013**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\text{m}\,\text{s}^{-2}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages and **3** blank pages.

1 A particle $\boldsymbol{P}$ is moving in a fixed circle of radius 0.5 m. At time $\boldsymbol{t}$ s its speed is $(2 - 2\boldsymbol{t} + \boldsymbol{t}^2)$ m s^{-1}. Find the magnitude of each of the radial and transverse components of the acceleration of $\boldsymbol{P}$ when $\boldsymbol{t} = 3$. [3]

2 Three uniform small smooth spheres $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$, of equal radii and of masses $4\boldsymbol{m}$, $\lambda\boldsymbol{m}$ and $\boldsymbol{m}$ respectively, are at rest in a straight line on a smooth horizontal plane, with $\boldsymbol{B}$ between $\boldsymbol{A}$ and $\boldsymbol{C}$. Sphere $\boldsymbol{A}$ is projected directly towards $\boldsymbol{B}$ with speed $\boldsymbol{u}$. The coefficient of restitution between $\boldsymbol{A}$ and $\boldsymbol{B}$, and between $\boldsymbol{B}$ and $\boldsymbol{C}$, is $\frac{1}{2}$. Show that the speed of $\boldsymbol{B}$ after it is struck by $\boldsymbol{A}$ is $\dfrac{6\boldsymbol{u}}{\lambda + 4}$. [4]

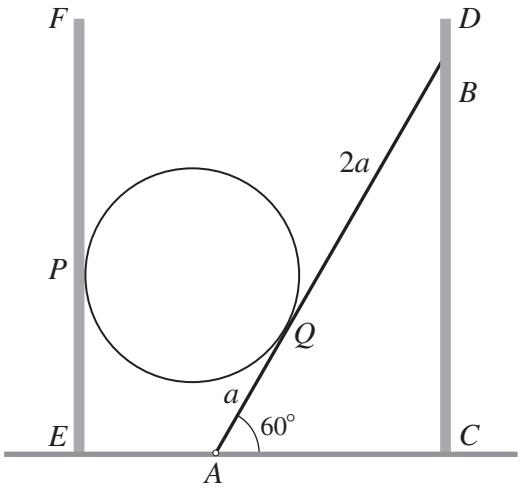
Given that the speed of $\boldsymbol{C}$ after it is struck by $\boldsymbol{B}$ is $\boldsymbol{u}$, find the value of λ. [5]

3 A particle $\boldsymbol{P}$ of mass $\boldsymbol{m}$ is attached to one end of a light inextensible string of length $\boldsymbol{a}$. The other end of the string is attached to a fixed point $\boldsymbol{O}$. The path of the particle is a complete vertical circle with centre $\boldsymbol{O}$. When $\boldsymbol{P}$ is at its lowest point, its speed is $\boldsymbol{u}$. When $\boldsymbol{P}$ is at the point $\boldsymbol{A}$, the tension in the string is $\boldsymbol{T}$ and the string makes an angle θ with the downward vertical, where $\cos\theta = \frac{3}{5}$. When $\boldsymbol{P}$ is at the point $\boldsymbol{B}$, above the level of $\boldsymbol{O}$, the tension in the string is $\frac{1}{8}\boldsymbol{T}$ and angle $\boldsymbol{BOA} = 90°$. Find $\boldsymbol{u}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{g}$. [9]

4 A particle $\boldsymbol{P}$ of mass $\boldsymbol{m}$ is attached to one end of a light elastic string of natural length $4\boldsymbol{a}$. The other end of the string is attached to a fixed point $\boldsymbol{O}$. The particle rests in equilibrium at the point $\boldsymbol{E}$, vertically below $\boldsymbol{O}$, where $\boldsymbol{OE} = 5\boldsymbol{a}$. The particle is pulled down a vertical distance $\frac{1}{2}\boldsymbol{a}$ from $\boldsymbol{E}$ and released from rest. Show that the motion of $\boldsymbol{P}$ is simple harmonic and state the period of the motion. [6]

Find the two possible values of the distance $\boldsymbol{OP}$ when the speed of $\boldsymbol{P}$ is equal to one half of its maximum speed. [4]

5



Two parallel vertical smooth walls ***EF*** and ***CD*** meet a horizontal plane at ***E*** and ***C*** respectively. A uniform smooth rod ***AB***, of weight $2\boldsymbol{W}$ and length $3\boldsymbol{a}$, is freely hinged to the horizontal plane at the point ***A***, between ***E*** and ***C***. The end ***B*** rests against ***CD***. A uniform smooth circular disc of weight ***W*** is in contact with the wall ***EF*** at the point ***P*** and with the rod at the point ***Q***. It is given that angle ***BAC*** is 60° and that $\boldsymbol{AQ} = \boldsymbol{a}$ (see diagram). The rod and the disc are in equilibrium in the same vertical plane, which is perpendicular to both walls. Show that

(i) the magnitude of the reaction at ***P*** is $\sqrt{3}\,\boldsymbol{W}$, [3]

(ii) the magnitude of the reaction at ***B*** is $\frac{7\sqrt{3}}{9}\boldsymbol{W}$. [4]

Find, in the form $\boldsymbol{kW}$, the magnitude of the reaction on ***AB*** at ***A***, giving the value of ***k*** correct to 3 significant figures. [5]

6 The random variable ***T*** is the time, in suitable units, between two successive arrivals in a hospital casualty department. The probability density function of ***T*** is f, where

$$\mathrm{f}(\boldsymbol{t}) = \begin{cases} 0.2\mathrm{e}^{-0.2\boldsymbol{t}} & \boldsymbol{t} \geqslant 0, \\ 0 & \text{otherwise.} \end{cases}$$

State the expected value of ***T***. [1]

Write down the distribution function of ***T*** and find $\mathrm{P}(\boldsymbol{T} > 10)$. [4]

7 Two independent random variables ***X*** and ***Y*** have distributions with the same variance σ^2. Random samples of ***n*** observations of ***X*** and $2\boldsymbol{n}$ observations of ***Y*** are taken and the results are summarised by

$$\Sigma\boldsymbol{x} = 10.0, \qquad \Sigma\boldsymbol{x}^2 = 25.0, \qquad \Sigma\boldsymbol{y} = 15.0, \qquad \Sigma\boldsymbol{y}^2 = 43.5.$$

Given that the pooled estimate of σ^2 is 2, find the value of ***n***. [7]

8 A factory produces china mugs. Random samples of size 6 are selected at regular intervals, and the mugs are inspected for defects. During one week, 100 samples are selected and the numbers of defective mugs found are summarised in the following table.

Number of defective mugs	0	1	2	3	4	5	6
Number of samples	11	43	35	8	2	1	0

Fit a binomial distribution to the data and carry out a goodness of fit test at the 5% significance level. [10]

9 A random sample of 9 observations of a normally distributed random variable $\boldsymbol{X}$ gave the following summarised data.

$$\Sigma \boldsymbol{x} = 94.5 \qquad \Sigma \boldsymbol{x}^2 = 993.6$$

Test, at the 5% significance level, whether the population mean of $\boldsymbol{X}$ is 10.2. [7]

Calculate a 90% confidence interval for the population mean of $\boldsymbol{X}$. [3]

10 The lengths, $\boldsymbol{x}$ m, and masses, $\boldsymbol{y}$ kg, of 12 randomly chosen babies born at a particular hospital last year are summarised as follows.

$$\Sigma \boldsymbol{x} = 7.50 \qquad \Sigma \boldsymbol{x}^2 = 4.73 \qquad \Sigma \boldsymbol{y} = 38.6 \qquad \Sigma \boldsymbol{y}^2 = 124.84 \qquad \Sigma \boldsymbol{xy} = 24.25$$

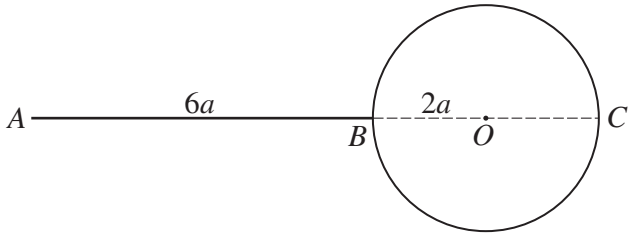
Find the value of the product moment correlation coefficient for this sample. [3]

Obtain an estimate for the mass of a baby, born last year at the hospital, whose length is 0.64 m. [4]

Test, at the 2% significance level, whether there is non-zero correlation between the two variables. [4]

11 Answer only **one** of the following two alternatives.

EITHER



A rigid body consists of a thin uniform rod $\boldsymbol{AB}$, of mass $4\boldsymbol{m}$ and length $6\boldsymbol{a}$, joined at $\boldsymbol{B}$ to a point on the circumference of a uniform circular disc, with centre $\boldsymbol{O}$, mass $8\boldsymbol{m}$ and radius $2\boldsymbol{a}$. The point $\boldsymbol{C}$ on the circumference of the disc is such that $\boldsymbol{BC}$ is a diameter and $\boldsymbol{ABC}$ is a straight line (see diagram). The body rotates about a smooth fixed horizontal axis through $\boldsymbol{C}$, perpendicular to the plane of the disc. The angle between $\boldsymbol{CA}$ and the downward vertical at time $\boldsymbol{t}$ is denoted by θ.

(i) Given that the body is performing small oscillations about the downward vertical, show that the period of these oscillations is approximately $16\pi\sqrt{\left(\dfrac{\boldsymbol{a}}{11\boldsymbol{g}}\right)}$. [9]

(ii) Given instead that the body is released from rest in the position given by $\cos\theta = 0.6$, find the maximum speed of $\boldsymbol{A}$. [5]

OR

Guided tours of a museum begin every 60 minutes. A randomly chosen tourist arrives $\boldsymbol{X}$ minutes after the start of a tour. The continuous random variable $\boldsymbol{X}$ has probability density function f given by

$$\mathrm{f}(\boldsymbol{x}) = \begin{cases} \dfrac{(\boldsymbol{x}-20)^2}{24\,000} & 0 < \boldsymbol{x} < 60, \\ 0 & \text{otherwise.} \end{cases}$$

The random variable $\boldsymbol{T}$ is the time that the tourist has to wait for the next tour to begin. Show that the distribution function G of $\boldsymbol{T}$ is given by

$$\mathrm{G}(\boldsymbol{t}) = \begin{cases} 0 & \boldsymbol{t} \leqslant 0, \\ \dfrac{8}{9} - \dfrac{(40-\boldsymbol{t})^3}{72\,000} & 0 < \boldsymbol{t} < 60, \\ 1 & \boldsymbol{t} \geqslant 60. \end{cases}$$ [5]

Find the median and the mean of $\boldsymbol{T}$. [9]

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